$S \circ R = \{(a,d),(b,d),(c,a),(d,b),(d,c),(e,d)\}$

$W_0 = \begin{bmatrix} 0&0&0&1&0 \\ 0&0&0&1&0 \\ 1&0&0&0&0 \\ 0&1&1&0&0 \\ 0&0&0&1&0 \end{bmatrix}$ $W_1 = \begin{bmatrix} 0&0&0&1&0 \\ 0&0&0&1&0 \\ 1&0&0&1&0 \\ 0&1&1&0&0 \\ 0&0&0&1&0 \end{bmatrix}$ $W_2 = \begin{bmatrix} 0&0&0&1&0 \\ 0&0&0&1&0 \\ 1&0&0&1&0 \\ 0&1&1&1&0 \\ 0&0&0&1&0 \end{bmatrix}$ $W_3 = \begin{bmatrix} 0&0&0&1&0 \\ 0&0&0&1&0 \\ 1&0&0&1&0 \\ 1&1&1&1&0 \\ 0&0&0&1&0 \end{bmatrix}$ $W_4 = \begin{bmatrix} 1&1&1&1&0 \\ 1&1&1&1&0 \\ 1&1&1&1&0 \\ 1&1&1&1&0 \end{bmatrix} = W_5$

北京邮电大学 2019—2020 学年

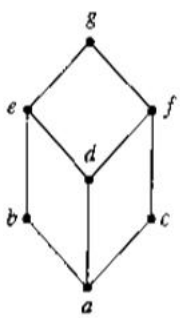
Discrete Mathematics – Final Examination (Paper B)

考试注意事项	
考	一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。
试	二、书本、参考资料、书包等与考试无关的东西一律放到考场指定位置。
注意	三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。
事项	四、学生必须将答题内容做在试题答卷上，做在草稿纸上一律无效。

考试课程	离散数学				考试时间				2020 年 6 月 30 日			
题号	一	二	三	四	五	六	七	八	九	十			总分
满分	10	10	10	14	6	10	10	10	10	10			
得分													
阅卷教师													

1. [10 points] Suppose $A=\{a,b,c,d,e\}$. Let R and S be the relations on A *described by* $R=\{(a,c), (b,c),(c,e), (d,a), (d,b),(e,c)\}$ *and* $S=\{(a,b),(a,c),(c,d),(d,e),(e,a)\}$. Use Warshall's algorithm to compute the transitive closure of $S \circ R$.

2. [10 points] Answer these questions for the partial order represented by this Hasse diagram.



上 下

c: f, g a

a:

   a) Compute LUB($\{b,c\}$). g
   b) Compute GLB($\{f, b\}$). a
   c) Is the poset a lattice? Explain your answer. √

任意对元素有最小上界.最大下界

3. [10 points] *Prove that if (G,*) and (G',*') are Abelian groups, then (G × G',*'') is an Abelian Group.*

$(G,*),(G',*') \rightarrow (a_1,b_1)*''(a_2,b_2) = (a_1*a_2, b_1*'b_2)$

$= (a_2*a_1, b_2*'b_1) = (a_2,b_2)*''(a_1,b_1)$

?

4. [14 points] Let m=3, n=6, H= $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix

$a)\ \begin{bmatrix} 0&0&0 \\ 0&0&1 \\ 0&1&0 \\ 0&1&1 \\ 1&0&0 \\ 1&0&1 \\ 1&1&0 \\ 1&1&1 \end{bmatrix}_{8\times 3} * \begin{bmatrix} 1&0&0&1&1&0 \\ 0&1&0&1&0&1 \\ 0&0&1&0&1&1 \end{bmatrix}_{3\times 6} = \begin{bmatrix} 0&0&0&0&0&0 \\ 0&0&1&0&1&1 \\ 0&1&0&1&0&1 \\ 0&1&1&1&1&0 \\ 1&0&0&1&1&0 \\ 1&0&1&1&0&1 \\ 1&1&0&0&1&1 \\ 1&1&1&0&0&0 \end{bmatrix}_{8\times 6}$

a) Determine the (3, 6) group code e_H: $B^3 \to B^6$.
b) Find the minimal distance of e_H.
c) How many errors will e_H detect ?
d) Decode the following words relative to a maximum likelihood decoding function associated with e_H.
(1) 101011 (2) 111011 (3) 000111

b) $\min(e_H) = |1001011| = 3$

c) $3 = k+1 \Rightarrow k=2$

d) (1) $\min\{\delta(101011, e(b))\} = \delta(101011, 001011) = 1$
$\Rightarrow d(101011) = 001$

(2) $110011 \Rightarrow 110$

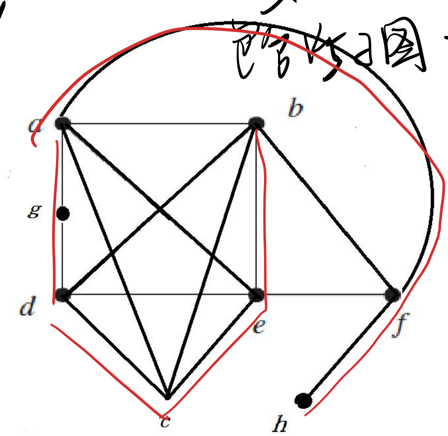
(3) $001011 \Rightarrow 001$??
$110011 \Rightarrow 110$

5. [6 points] Determine whether the given graphs have an Euler circuit or a Hamilton circuit. If not, determine whether the given graphs have an Euler path or a Hamilton path. And determine whether the given graphs are planar.

~~欧=哈~~ × × × ✓

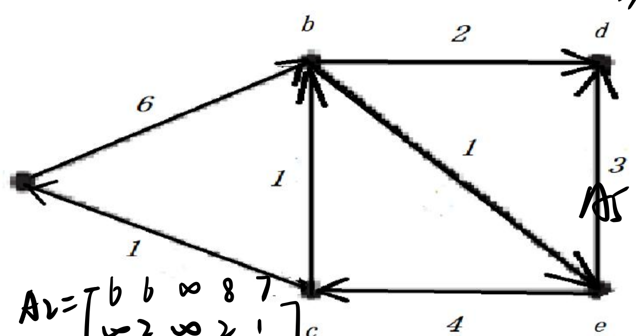
hf

路哈图



6. [10 points] Finding the shortest path length between any two vertices using distance matrix.

$A_0 = \begin{bmatrix} \infty&6&\infty&\infty&\infty \\ \infty&\infty&\infty&2&1 \\ 1&1&\infty&\infty&\infty \\ \infty&\infty&\infty&\infty&\infty \\ \infty&\infty&4&3&\infty \end{bmatrix}$



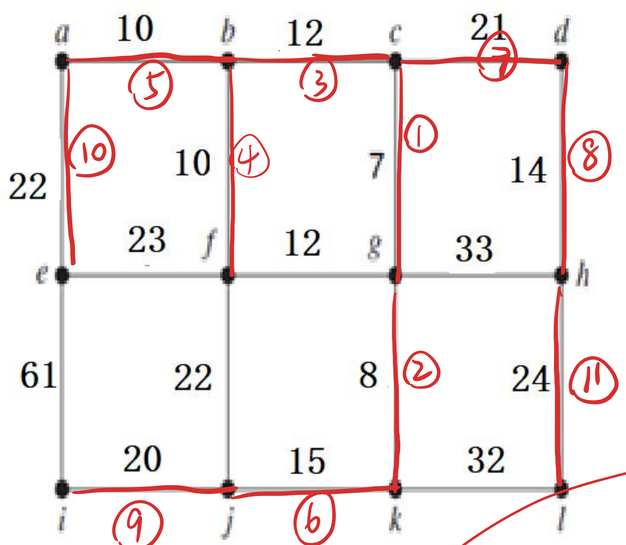
$A_1 = \begin{bmatrix} 6&6&\infty&\infty&\infty \\ \infty&\infty&\infty&2&1 \\ 1&1&\infty&\infty&\infty \\ \infty&\infty&\infty&\infty&\infty \\ \infty&\infty&4&3&\infty \end{bmatrix}$

$A_2 = \begin{bmatrix} 6&6&\infty&8&7 \\ \infty&2&\infty&2&1 \\ 1&1&\infty&3&2 \\ \infty&\infty&\infty&\infty&\infty \\ \infty&\infty&4&3&\infty \end{bmatrix}$

$A_3 = \begin{bmatrix} 6&6&\infty&8&7 \\ \infty&2&\infty&2&1 \\ 1&1&5&3&2 \\ \infty&\infty&\infty&\infty&\infty \\ 5&5&4&3&6 \end{bmatrix} = A_4$

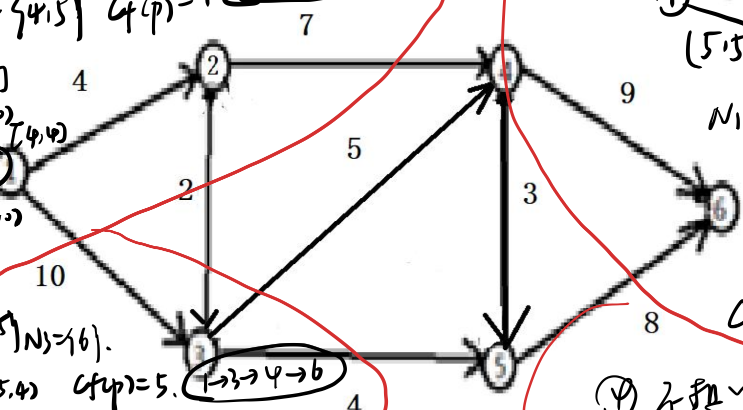
$A_5 = \begin{bmatrix} 6&6&11&8&7 \\ 6&2&5&2&1 \\ 1&1&5&3&2 \\ \infty&\infty&\infty&\infty&\infty \\ 5&5&4&3&4 \end{bmatrix}$

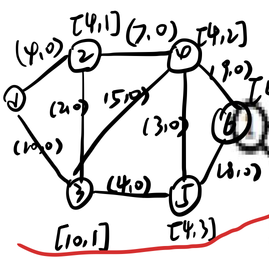
7. [10 points] Use Kruskal's algorithm to design a minimum-cost communications network connecting all the computers represented by the graph in next Figure.



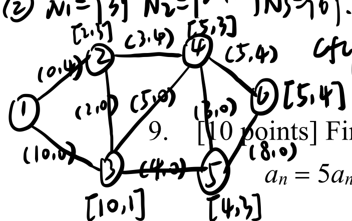
8. [10 points] Find a maximum flow in the given network by using the labeling algorithm. And find a minimum cut of this network. (Please give out the labeling graph of every flow.)

① $N_1=\{2,3\}$ $N_2=\{4,5\}$ $c(p)=4$ $1\to2\to4\to6$

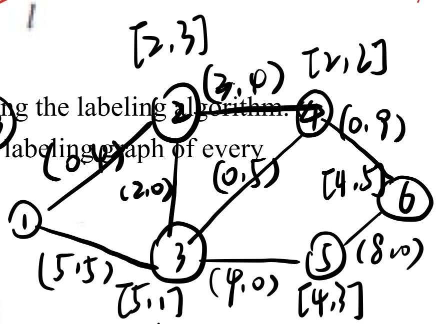




② $N_1=\{3\}$ $N_2=\{2,4,5\}$ $N_3=\{6\}$. $c(p)=5$. $1\to3\to4\to6$



③ $N_1=\{3\}$ $N_2=\{2,5\}$ $N_3=\{4,6\}$ $c(p)=4$. $1\to3\to5\to6$



④ 不想写了…

9. [10 points] Find the solution to the recurrence relations

$$a_n = 5a_{n-1} - 4a_{n-2},\ a_0 = 1,\ a_1 = 9.$$

$r^2-5r+4=0 \Rightarrow r_1=1,\ r_2=4 \Rightarrow a_n=\alpha_1+\alpha_2 4^n \Rightarrow \begin{cases}\alpha_1+\alpha_2=1\\ \alpha_1+4\alpha_2=9\end{cases} \Rightarrow \begin{cases}\alpha_1=-\frac{5}{3}\\ \alpha_2=\frac{8}{3}\end{cases}$

10. [10 points] Set up a generating function and use it to find the number of ways in which eleven identical coins can be put in three distinct envelopes (labeled A, B, C) if envelope A has at least three coins in it.

$A+B+C=11$

$A\ge3 \Rightarrow 3\le A\le 11$

$0\le B\le 8$ $0\le C\le 8$

$(x^3+x^4+\cdots+x^{11})(1+x+\cdots+x^8)^2$

$= x^3(1+x+\cdots+x^8)^3 \Rightarrow$ 找 x^8 系数

~~$1+2+3+\cdots$~~ $\Rightarrow 9$