[10 points] In the questions below, describe each sequence recursively. Include initial conditions and assume that the sequences begin with a_1.

a) $a_n = 5^n$

$a_1 = 5$

$a_{n+1} = 5a_n$

$r - C_2 = 0$

$a_1 r_1^n + a_2 r_2^n$

...101, 1010101, ...

$a_1 = 1$

$a_n = a_1 r_1^n + a_2 r_2^n$

b) 1, 101, 10101, 1010101, ...

$a_1 = 1$

$a_{n+1} = (10^{n+1})^2 + a_n$

c) a_n = the number of bit strings of length n with an even number of 0s

n

$a_1 =$

a_{n-1}

$a_n = (2^{n-1} - a_{n-1}) +$

$n-1$

1

a_n

d) a_n = the number of ways to go down an n-step staircase if you go down 1, 2, or 3 steps at a time.

$a_{n+1} = a_n + a_{n-1} + a_{n-2}$

$a_1 = 1$

$a_2 = 2$

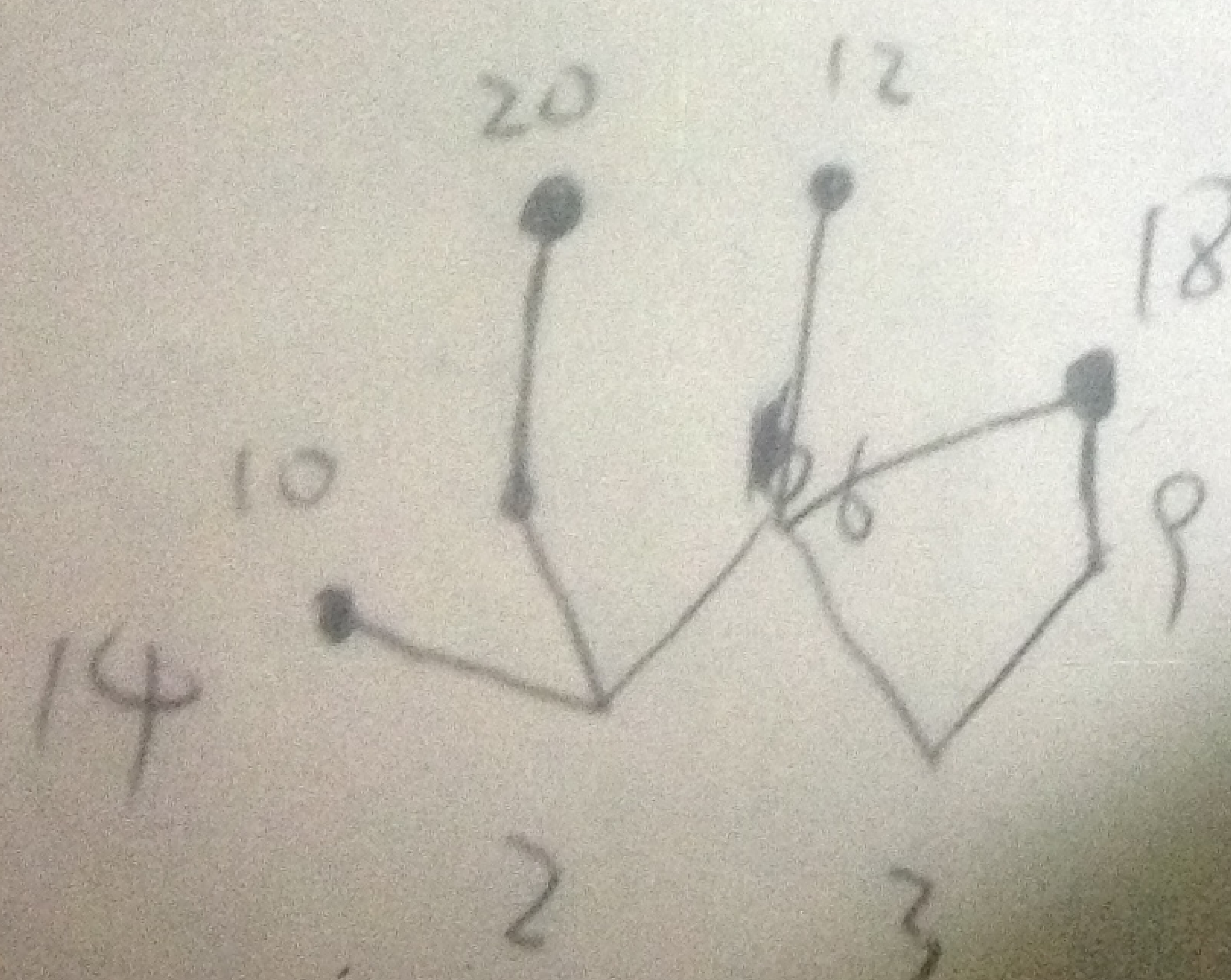
$a_3 = 4$

$a_4 = 7$

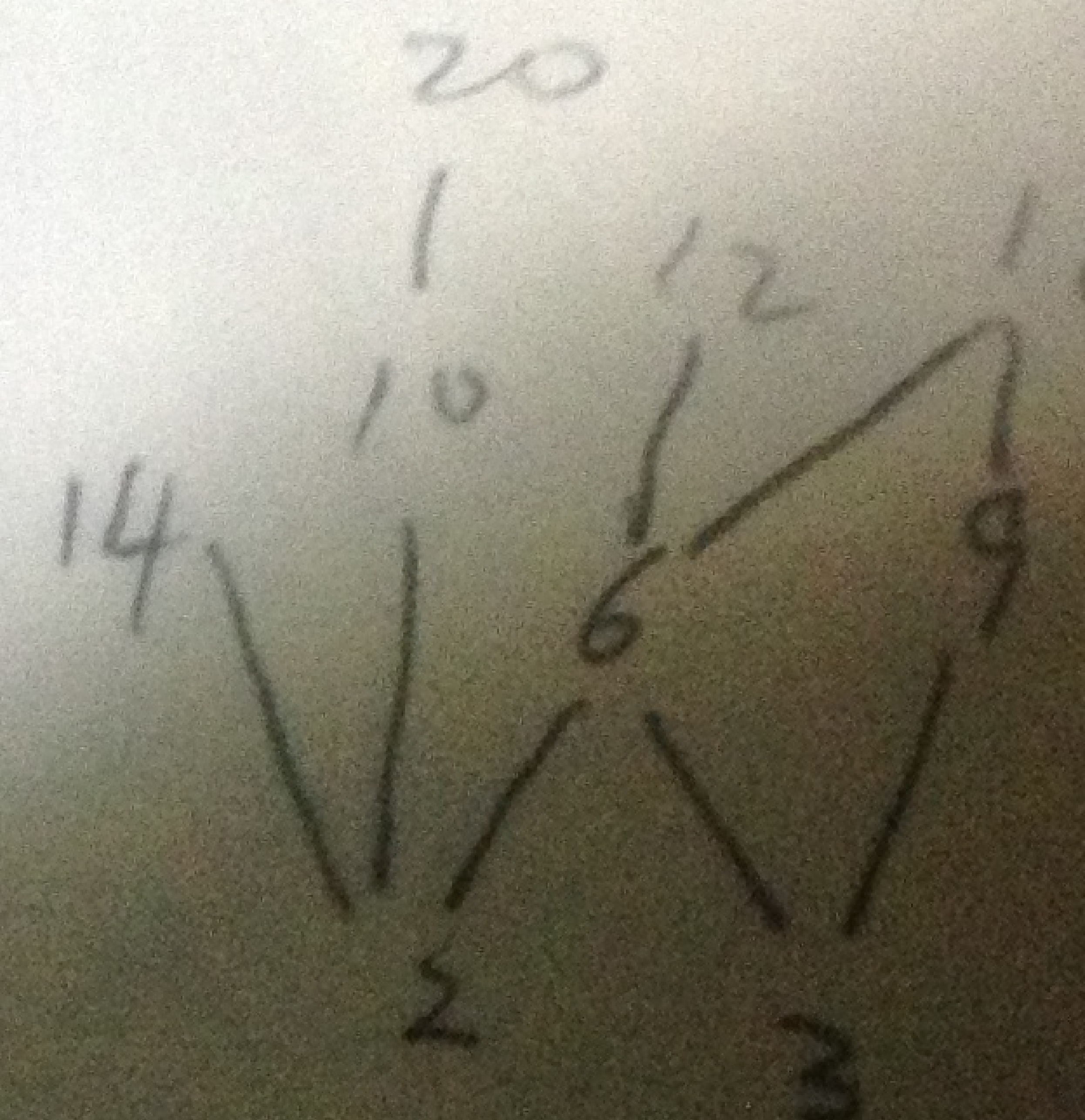
$a_5 =$

[10 points] Suppose $A = \{2,3,6,9,10,12,14,18,20\}$ and R is the partial order relation defined on A where xRy means x is a divisor of y.

a) Draw the Hasse diagram for R.



9



14

b) Find all maximal elements.

14 20 12

c) Find all minimal elements.

2 3

d) Find lub({3,10})

e) Find glb({14,10})

φ : 在集合中 $a \neq b$

4. [10 points] In the questions below give an example or else prove that there are none:

a) A relation on $\{a,b,c\}$ that is reflexive and transitive, but not antisymmetric.

自反 传递 反对称

$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\{(a,a)\ (b,b)\ (c,c)\ (a$

R $R^2 \subset R$, $R^3 \subset R$ $R^n \subset R$

b) ~~A relation on {1,2}~~ that is symmetric and transitive, but not reflexive.

对称、传递、自反

$\{1,1\}$ $\{1,2\}$ $\{2,1\}$

$(1,1)$ $(1,2)$

$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

c) A relation on $\{1,2,3\}$ that is reflexive and transitive, but not symmetric.

自反 传递 对称

$(1,1)\ (2,2)\ (1,2)\ (2,1)$

(1,1) (1,2)

c) A relation on {1,2,3} that is reflexive and transitive, but not symmetric.

自反 传递 对称

(1,1) (2,2) (1,2) (2,1) (1,3) (3,3)

edge

5. [10 points] In the questions below fill in the blanks.

a) W_n has ~~2n~~ edges and ~~n(n-1)~~ vertices.

n+1

b) The adjacency matrix for K_n has n(n-1) 1s and n 0s.

c) If G is a connected graph with 12 regions and 20 edges, then G has 10 vertices.

$r = e - v + 2$

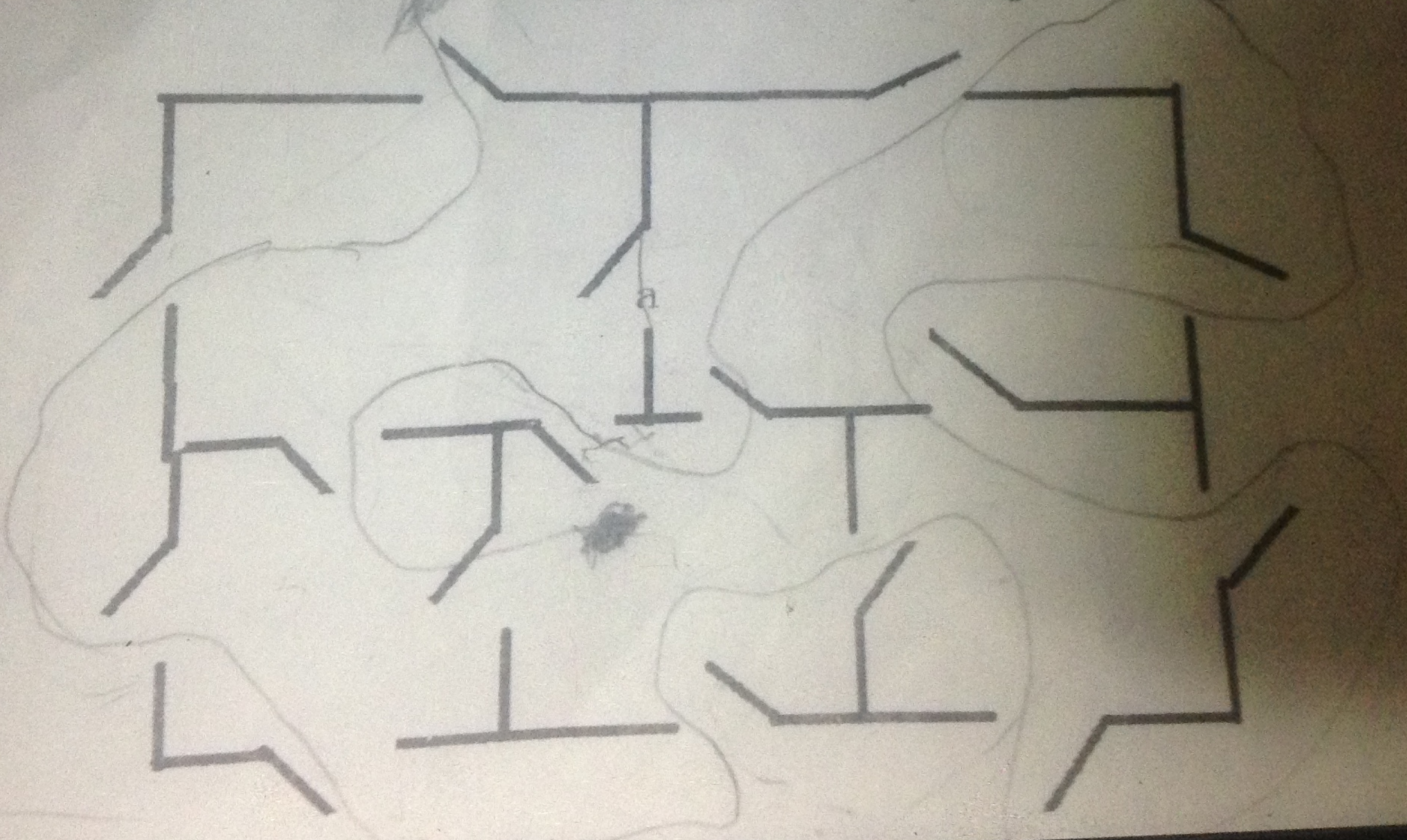
d) The vertex-chromatic number for $K_{7,7}$ = 2.

e) If a regular graph G has 10 vertices and 45 edges, then each vertex of G has degree 9.

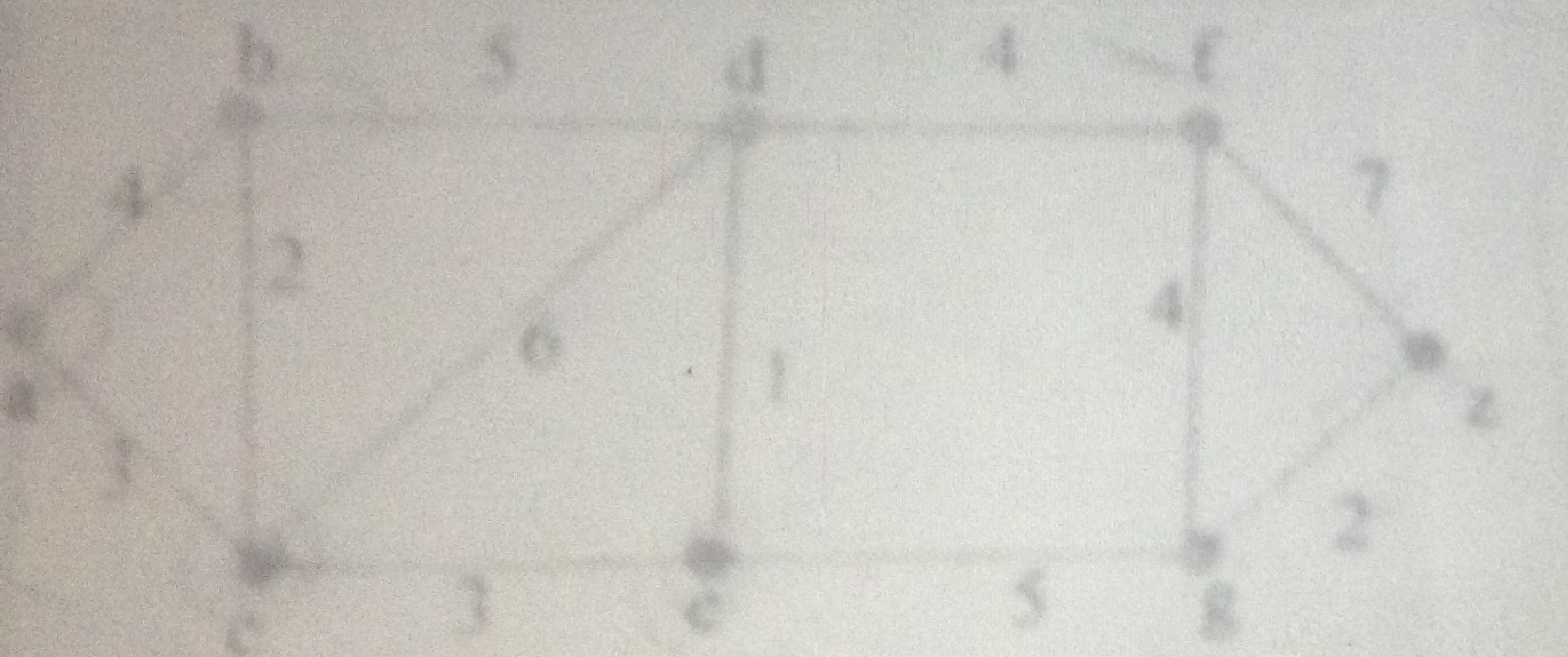
5. [10 points] An old puzzle presents a house with 5 rooms and 16 doors, as shown in the following figure. The problem is to figure out how to begin in a room or outside and take a walk that goes through each door exactly once.

a) Is such a walk possible? Explain.

b) How does your answer change if the door "*a*" adjioning the two large rooms is closed?



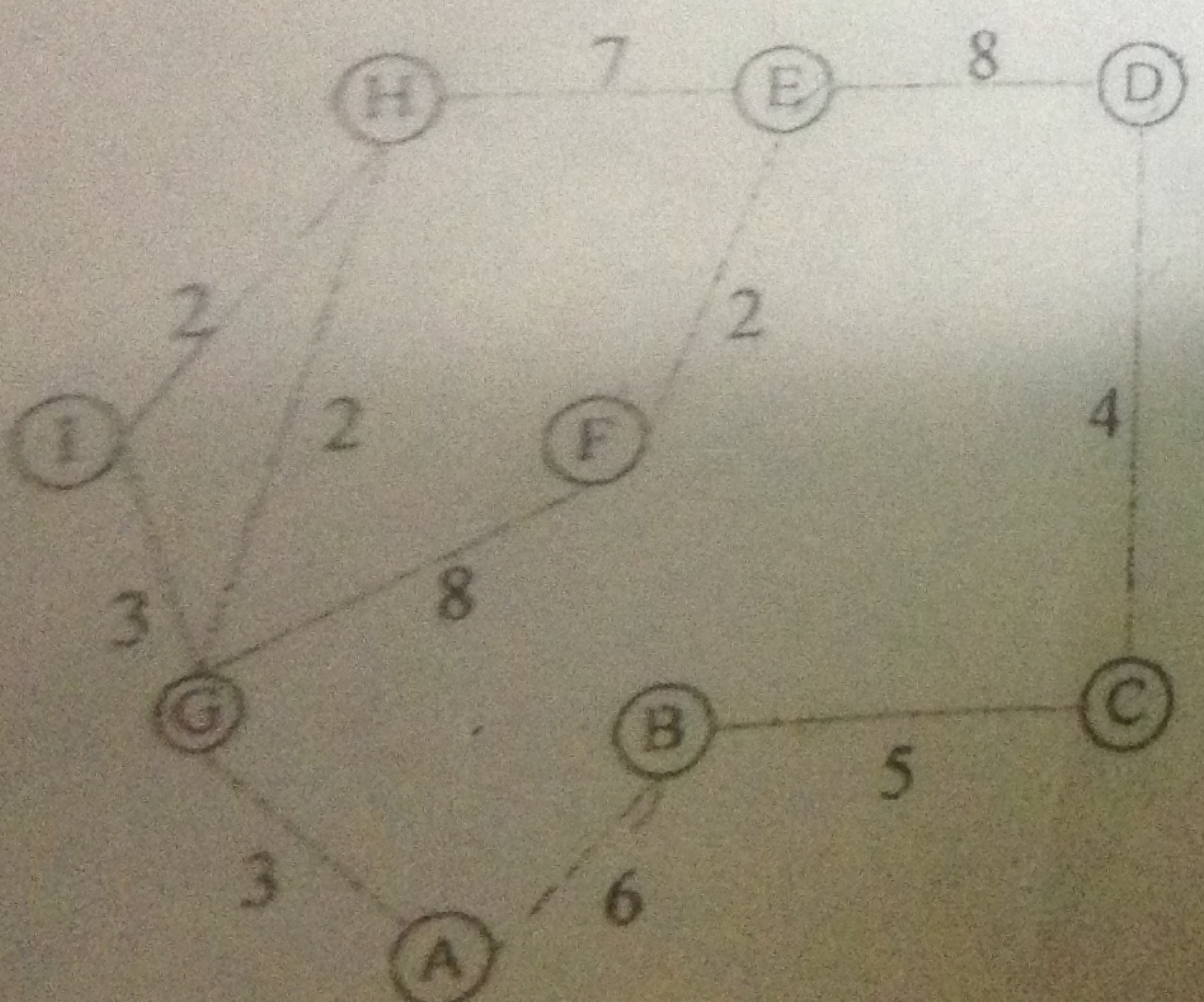
[10 points] Use Dijkstra's Algorithm to find the shortest path length between the vertices a and z in this weighted graph. (Please give the process!)



[10 points] Find a minimal spanning tree for the relation given by the graph.

a) Use Prim's algorithm, Start from node H. (Write down the detail process.)

b) Use Kruskal's algorithm. List the edges in the order in which they are chosen.



S={x,y,z,w}

[10 points] Let $(S,*)$ be the semigroup whose operation table is give below. Let R be the equivalence relation on S defined by the partition $\{\{x,y\},\{z,w\}\}$. Show that R is a congruence relation on $(S,*)$, and construct the operation table for quotient semigroup($S/R,\odot$).

*	x	y	z	w
x	x	y	z	w
y	y	z	w	z
z	z	z	z	z
w	w	w	w	w

commutative 可交换的

associative 可结合的

idempotent 幂等

[x]=[y]

[z]=[w]

$x*y=$

$z*w=$

等价类 S×T

$(S,*)(T,*')$

S×T=

xRy

zRw