

# 离散数学

Discrete mathematics

1. 这题估计是被某种神秘的力量所吞噬了，所以请自己猜猜这道题到底考了什么。
2. [10 points] In the questions below, describe each sequence recursively. Include initial conditions and assume that the sequences begin with  $a_1$ .
  - a)  $a_n = 5^n$
  - b) 1, 101, 10101, 1010101 ... ..
  - c)  $a_n$  = the number of bit strings of length  $n$  with an even number of 0s.
  - d)  $a_n$  = the number of ways to go down an  $n$ -step staircase if you go down 1, 2, or 3 step at a time.

3. [10 points] Suppose  $A = \{2, 3, 6, 9, 10, 12, 14, 18, 20\}$  and  $R$  is the partial order relation defined on  $A$  where  $xRy$  means  $x$  is a divisor of  $y$ .

- a) Draw the Hasse diagram for  $R$ .
- b) Find all maximal elements.
- c) Find all minimal elements.
- d) Find  $\text{lub}(\{3, 10\})$
- e) Find  $\text{glb}(\{14, 10\})$

4. [10 points] In the questions below give an example or else prove that there are none.

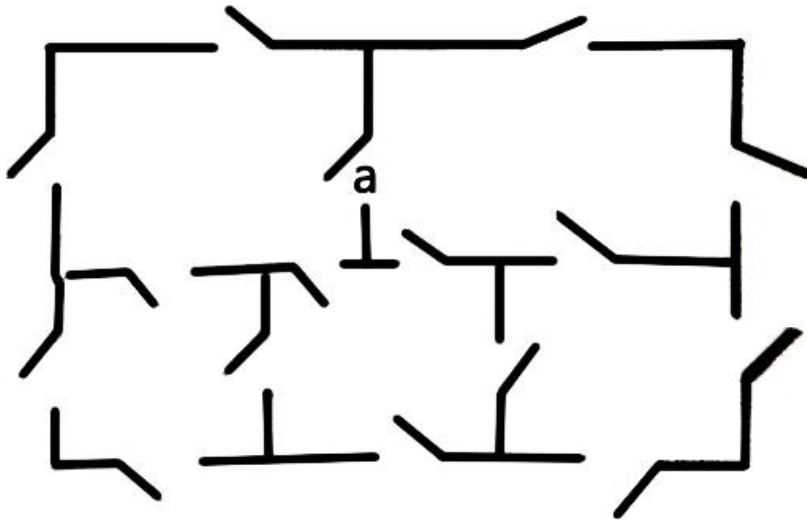
- a) A relation on  $\{a, b, c\}$  that is reflexive and transitive, but not antisymmetric.
- b) A relation on  $\{1, 2\}$  that is symmetric and transitive, but not reflexive.
- c) A relation on  $\{1, 2, 3\}$  that is reflexive and transitive, but not symmetric.

5. [10 points] In the questions below fill in the blanks.

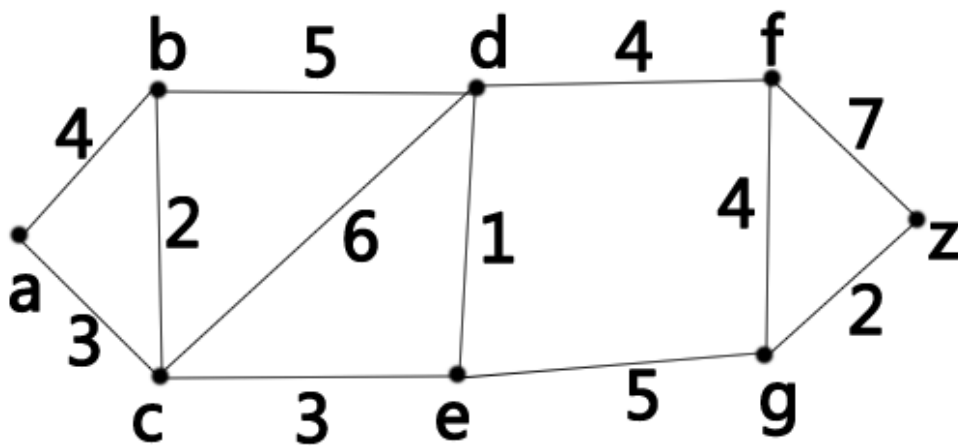
- a)  $W_n$  has \_\_\_\_\_ edges and \_\_\_\_\_ vertices.
- b) The adjacency matrix for  $K_n$  has \_\_\_\_\_ 1s and \_\_\_\_\_ 0s.
- c) If  $G$  is a connected graph with 12 regions and 20 edges, then  $G$  has \_\_\_\_\_ vertices.
- d) The vertex-chromatic number for  $K_{7,7}$  = \_\_\_\_\_.
- e) If a regular graph  $G$  has 10 vertices and 45 edges, then each vertex of  $G$  has degree \_\_\_\_\_.

6. [10 points] An old puzzle presents a house with 5 rooms and 16 doors, as shown in the following figure. The problem is to figure out how to begin in a room or outside and take a walk that goes through each door exactly once.

- a) Is such a walk possible? Explain.
- b) How does your answer change if the door "a" adjoining the two large rooms is closed?



7.[10 points] Use Dijkstra's Algorithm to find the shortest path length between the vertices a and z in this weighted graph.(Please give the process!)

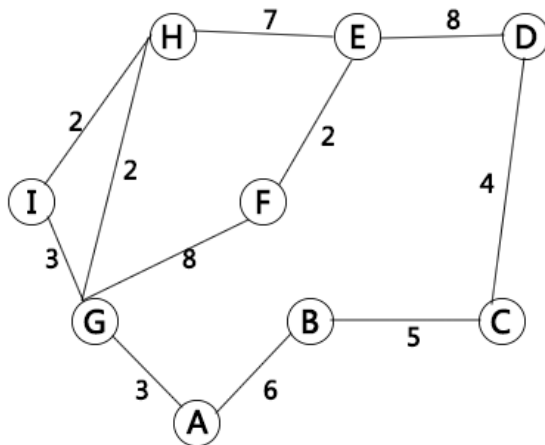


8.[10 points] Find a minimal spanning tree for the relation given by the graph .

a) Use Prim's algorithm, start from node H.(Write down

the detail process)

- b) Use Kruskal' s algorithm.List the edges in the order in which they are chosen.



9. [10 points] Let  $(S,*)$  be the semigroup whose operation table is given below.Let  $R$  be the equivalence relation on  $S$  defined by the partition  $\{\{x,y\},\{z,w\}\}$ .Show that  $R$  is a congruence relation on  $(S,*)$ ,and construct the operation table for quotient semigroup  $(S/R, \odot)$ .

$*$	$x$	$y$	$z$	$w$
$x$	$x$	$y$	$z$	$w$
$y$	$y$	$z$	$w$	$z$
$z$	$z$	$z$	$z$	$z$
$w$	$w$	$w$	$w$	$w$