离散の数学

Discrete mathematics

1. 这题估计是被某种神秘的力量所吞噬了，所以请自己猜猜这道题到底考了什么。

2. [10 points] In the questions below,describe each sequence recursively.Include intial conditions and assume that the sequences begin with a1.

a) $a_n=5^n$

b) 1,101,10101,1010101 … …

$a_n = 100a_n + 1$

$a_{n-1} + 1$ (0 / 1)

$a_{n-2} + 00$

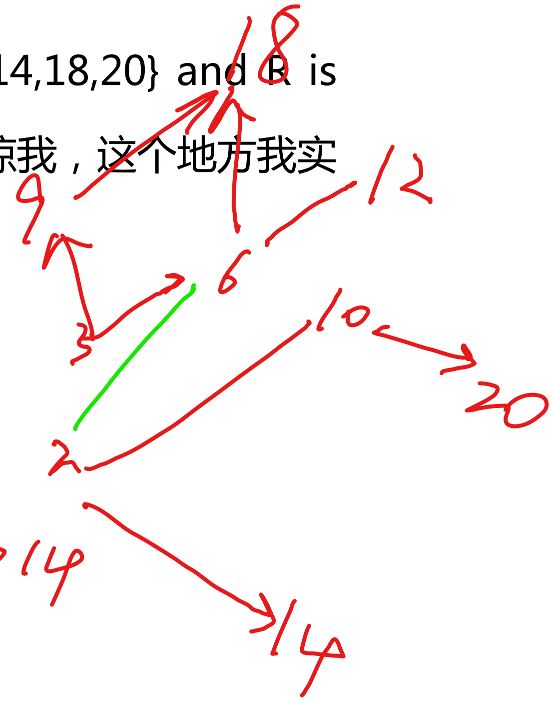
$+ 11$

c) a_n=the number of bit strings of length n with an even number of 0s.

d) a_n=the number of ways to go down an n-step staircase if you go down 1,2,or 3 step at a time.

3. [10 points]Suppose A={2,3,6,9,10,12,14,18,20} and R is the partial order relation balabalaba(原谅我，这个地方我实在看不清).

A where xRy means x is a divisor of y.



a) Draw the Hasse diagram for R.

b) Find all maximal elements. 12, 18, 20, 14

c) Find all minimal elements. 2 3

d) Find lub({3,10}) NO

e) Find glb({14,10}) 2

4. [10 points] In the questions below give an example or else prove that there are none.

a) A relation on {a,b,c} that is reflexive and transitive,but not antisymmetric.

	a	b	c
a	1	1	1
b	1	1	1
c	1	1	1

b) A relation on {1,2} that is symmetric and transitive,but not reflexive. none

c) A relation on {1,2,3} that is reflexive and transitive,but not symmetric.

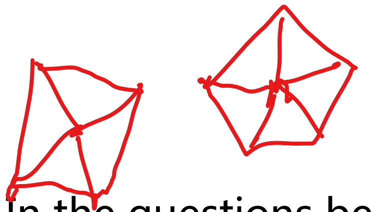
1→1

1→2

2→2

3→3

	1	2	3
1	1	1	0
2	0	1	0
3	0	0	1



5. [10 points] In the questions below fill in the blanks.

a) W_n has __ $2n$ __edges and __ $n+1$ __vertices.

b) The adjacency matrix for K_n has__ n^2-n __1s and __ n __0s.

$12=20-V+2$

c) If G is a connected graph with 12 regions and 20 edges,then G has __ 10 __vertices.

d) The vertex-chromatic number for $K_{7,7}$=__ 2 __.

e) If a regular graph G has 10 vertices and 45 edges,then each vertex of G has degree__ 9 __.

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6. [10 points] An old puzzle presents a house with 5 rooms and 16 doors,as shown in the following figure.The problem is to figure out how to begin in a room or outside and take a walk that goes through each door exactly once.

a) Is such a walk possible?Explain.

b) How does your answer change if the door “a” adjoining the two large rooms is closed?

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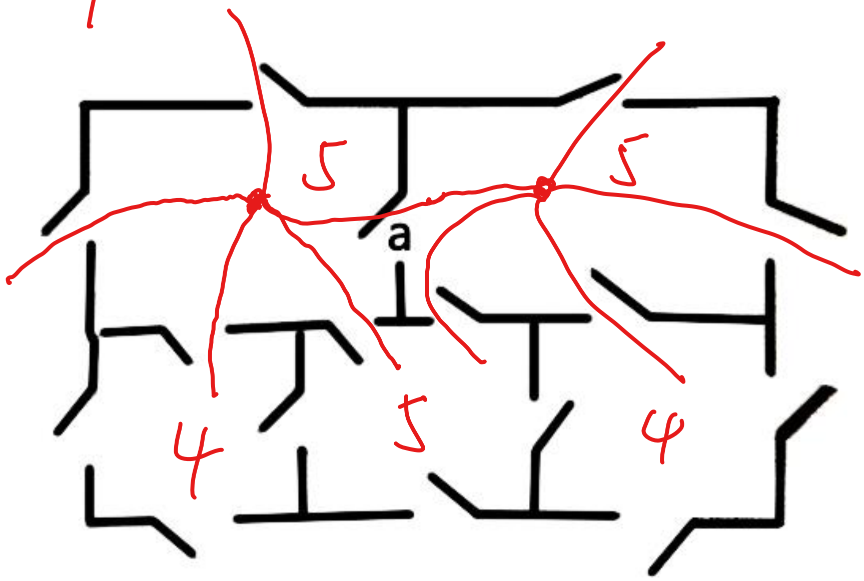
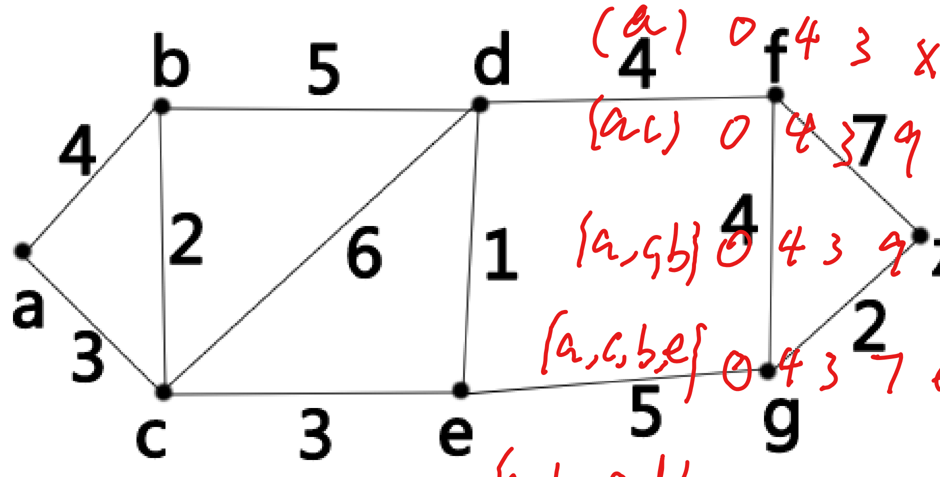


7.[10 points] Use Dijkstra’ s Algorithm to find the shortest path length between the vertices a and z in this weighted graph.(Please give the process!)



∅	a	b	c	d	e	f	g	z
	0	x	x	x	x	x	x	x
(a)	0	4	3	x	x	x	x	x
{a,c}	0	4	3	9	6	x	x	x
{a,c,b}	0	4	3	9	6	x	x	x
{a,c,b,e}	0	4	3	7	6	x	11	x
{a,b,c,e,d}	0	4	3	7	6	11	11	x
{a,b,c,d,e,f}	0	4	3	7	6	11	11	18
{∅ g	0	4	3	7	6	11	11	13

8.[10 points] Find a minimal spanning tree for the relation given by the graph .

a) Use Prim’ s algorithm,star from node H.(Write down

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the detail process)

b) Use Kruskal’ s algorithm.List the edges in the order in which they are chosen.

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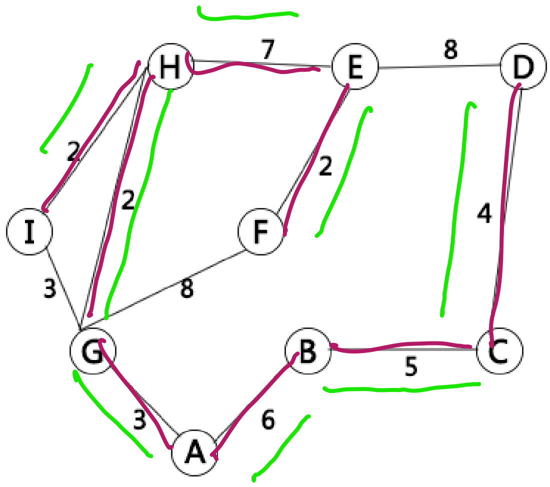
⊙⊙ $a*b=$

⊙

$[a]\odot[b]$

$=[a*b]$



9. [10 points] Let(S,*) be the semigroup whose operation table is given below.Let R be the equivalence relation on S defined by the partition{{x,y},{z,w}}.Show that R is a congruence relation on (S,*),and construct the operation table for quotient semigroup(S/R, ⊙).

错题!

aRb

cRd

$a*cRb*d$

*	x	y	z	w
x	x	y	z	w
y	y	z	w	z
z	z	z	z	z
w	w	w	w	w

yRy

xRy

$x*yRy*y$

yRz

xRy

zRw

$x*zRy*w$ z w

$x*wRy*z$

w z w

故R为S上的同余关系

⊙	[x]	[z]
[x]	[x]	[z]
[z]	[z]	[z]